

Example 2:

Image: (image2)

Natural language theorem statement: Let a , b , and c be the side lengths of a triangle with inradius r . If $p = \frac{a+b+c}{2}$, show that

$$\frac{1}{(p-a)^2} + \frac{1}{(p-b)^2} + \frac{1}{(p-c)^2} \geq \frac{1}{r^2}.$$

Output (Formal theorem using Lean4 language):

```
import Mathlib
```

```
theorem putnam_a2
```

```
(r : ℝ)
```

```
(A B C : EuclideanSpace ℝ (Fin 2))
```

```
(hABC : Collinear ℝ {A, B, C})
```

```
(a b c p : ℝ)
```

```
(ha : a = dist B C)
```

```
(hb : b = dist C A)
```

```
(hc : c = dist A B)
```

```
(hp : p = (dist B C + dist C A + dist A B)/2)
```

```
(hr : ∃ I : EuclideanSpace ℝ (Fin 2),
```

```
  (∃! P : EuclideanSpace ℝ (Fin 2), dist I P = r ∧ Collinear ℝ {P,  
    ↪ B, C}) ∧
```

```
  (∃! Q : EuclideanSpace ℝ (Fin 2), dist I Q = r ∧ Collinear ℝ {Q,  
    ↪ C, A}) ∧
```

```
  (∃! R : EuclideanSpace ℝ (Fin 2), dist I R = r ∧ Collinear ℝ {R,  
    ↪ A, B}) ∧
```

```
  (∀ Z : EuclideanSpace ℝ (Fin 2), dist I Z ≤ r → Z ∈ convexHull  
    ↪ ℝ {A, B, C}))
```

```
: 1/(p - a)^2 + 1/(p - b)^2 + 1/(p - c)^2 ≥ 1/r^2 :=
```

```
sorry
```

Strict Instructions:

- Only output the formal theorem in Lean 4, including all necessary imports, variable declarations, hypotheses, and the theorem statement.
- Do NOT include any proof or attempt to prove the theorem.
- Explicitly indicate that this is an unproven theorem by ending the statement with `:= sorry`.
- Do not use `by`, `exact`, or any other proof-related keywords.