Collective modes and screening in an electric-magnetic dual plasma

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We study the linear response of an effective relativistic two-fluid medium carrying separately conserved electric and magnetic charge currents. The model is defined by the duality-symmetric Maxwell equations with electric and magnetic sources, together with Lorentz-force dynamics for two fluids with independent inertia and possible Carter-type entrainment. The magnetic component is treated as an effective charge-carrying constituent, so the analysis uses only the closed two-fluid equations.

Around a homogeneous, neutral, and unmagnetized background, the transverse electromagnetic response contains two stable branches whose cutoffs are set by the electric and magnetic plasma frequencies and are exchanged by electric–magnetic duality. In the longitudinal sector, entrainment mixes the electric and magnetic density oscillations, turns their crossing into an avoided crossing, and gives the stability condition $\kappa^2 < 1$, equivalent to positive definiteness of the two-fluid momentum matrix. Resolving the magnetic component into monopole and antimonopole species gives a neutral branch selected by magnetic charge conjugation C_m. In this branch the net magnetic current vanishes, so the long-range monopole field is absent, while the total magnetic density can still produce screened collective response. The resulting picture is that magnetic charge can be statically hidden but dynamically visible. A robust observable signature is the density scaling $\omega_{\rm coll}^2 \sim \omega_{pm}^2 \propto n_{(m)}^0$, which may survive dissipative broadening even when sharp ideal-plasma poles are not resolved. We briefly comment on possible dyonic interpretations of magnetically neutral composites, but the linear-response results do not rely on that interpretation.

I. INTRODUCTION

Magnetic charge is usually discussed as an additional source of the Maxwell field. In the standard Dirac picture, a magnetic monopole is represented by a singular potential, and the unobservability of the Dirac string leads to charge quantization [1–3]. The same physics can be described globally using gauge patches in the Wu–Yang formulation [4]. Equivalent local descriptions, such as the two-potential formulation of Cabibbo and Ferrari, the local Zwanziger action, and the covariant PST construction, make electric–magnetic duality manifest at the price of an enlarged potential structure [5–7]. These formulations establish how magnetic charge can be included consistently in electrodynamics. The question addressed here is different and more phenomenological: if magnetic charge is carried by a dynamical medium with its own inertia, what collective electromagnetic response follows?

We therefore study an effective electric–magnetic two-fluid plasma. The minimal closed model consists of the duality-symmetric Maxwell equations with electric and magnetic sources,

$$\nabla_b F^{ab} = j^a_{(e)}, \qquad \nabla_b {}^*F^{ab} = j^a_{(m)}.$$

The two charge currents are carried by independently conserved electric and magnetic fluids, with charge densities $j^a_{(e)} = q n^a_{(e)}$ and $j^a_{(m)} = g n^a_{(m)}$. Their inertial response, including possible Carter-type entrainment, is specified in Sec. II A. No microscopic monopole core model is assumed; the magnetic component is treated as an effective charge-carrying constituent whose inertia is a low-energy parameter of the medium.

One motivation for this effective system comes from the convective variational, or matter-space, description of relativistic fluids and multifluids developed by Carter and subsequent authors [8–12], and from its application to electromagnetism and two-sector decompositions [13, 14]. In that framework conserved currents arise naturally from matter-space pull-backs, and a magnetic charge-carrying constituent can be represented at the effective level by a conserved magnetic current. The present paper does not require the details of that construction: it takes the closed electric–magnetic two-fluid equations as an effective model and computes their linear collective response.

We linearize the dual plasma around a static, homogeneous, neutral, and unmagnetized background. The central question is whether the magnetic carrier is dynamically inert when the background has no net monopole field, or whether it leaves observable signatures in the collective spectrum. The answer is that a magnetically neutral medium

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can be statically hidden but dynamically visible. In the transverse sector the response contains two duality-related plasma scales, set separately by the electric and magnetic plasma frequencies. The resulting dispersion relation has two stable branches; when the magnetic plasma frequency is taken to zero, one branch reduces to the ordinary electric plasma mode while the additional branch collapses. This identifies the extra mode as the collective response of the magnetic carrier.

The longitudinal sector exposes the two-fluid nature of the system more directly. Without entrainment the electric and magnetic density oscillations give two independent Langmuir modes. With entrainment, the off-diagonal inertial response mixes the two oscillations, turns their crossing into an avoided crossing, and yields the stability condition

$$\kappa^2 < 1,$$

equivalent to positive definiteness of the two-fluid momentum matrix.

We also examine a magnetically neutral branch obtained by resolving the magnetic component into monopole and antimonopole species. A magnetic charge-conjugation symmetry $\mathcal{C}_m$ selects a background with vanishing net magnetic current, so the long-range monopole field is absent even though the total magnetic density is nonzero. The remaining magnetic-charge fluctuations are screened, but they still leave a collective response controlled by the magnetic plasma frequency. Thus the most robust observable signature is the density scaling

$$\omega_{\rm coll}^2 \sim \omega_{pm}^2 \propto n_{(m)}^0,$$

which may survive dissipative broadening in effective monopole media.

We also briefly comment on a possible structural implication of the same $\mathcal{C}_m$-symmetric viewpoint. A pair of dyons with opposite magnetic charges can be magnetically neutral as a whole while carrying a nonzero electric charge. The mutual Dirac–Schwinger–Zwanziger pairing of the two constituents can then constrain the residual electric charge even though no free monopole appears in the asymptotic spectrum. This observation is not used in the linear plasma calculation and should be regarded as a possible interpretive extension rather than as an input to the mode analysis.

The paper is organized as follows. In Sec. II we derive the linear transverse and longitudinal modes of the dual plasma in the absence of entrainment. In Sec. III we include entrainment, derive the longitudinal avoided crossing and stability bound, analyze the $\mathcal{C}_m$-protected neutral magnetic branch, and discuss screening and thermal de-screening. Section IV summarizes the physical interpretation: static hiding of net magnetic charge, dynamical visibility through collective response, and possible extensions to effective monopole media, cosmology, and dyonic composites.

II. ELECTROMAGNETIC MODES OF THE DUAL PLASMA

We first examine the linear electromagnetic response of the dual plasma before including entrainment. This already shows that the medium is not merely a reparametrization of conventional monopole magnetohydrodynamics. The magnetic fluid supplies an independent dynamical response to the magnetic field, in the same sense that the electric fluid supplies the usual plasma response to the electric field. As a result, the electromagnetic spectrum contains two plasma scales and two propagating transverse branches.

We linearize the equations about a static, homogeneous, charge-neutral, and unmagnetized background,

$$\mathbf{E}_0 = \mathbf{B}_0 = 0, \qquad n_{(e)} = n_{(e)}^0, \qquad n_{(m)} = n_{(m)}^0,$$

with both fluids initially at rest. Perturbations are taken to be plane waves $\propto \exp[i(\mathbf{k}\cdot\mathbf{x} - \omega t)]$. Throughout this section we work in the cold limit and neglect direct entrainment; the entrainment-induced longitudinal mixing is treated in Sec. III.

A. Setup: the closed two-fluid model

We take as our starting point an effective closed two-fluid model with separately conserved electric and magnetic charge currents. The model is defined by the duality-symmetric Maxwell equations

$$\nabla_b F^{ab} = j_{(e)}^a, \qquad \nabla_b {}^*F^{ab} = j_{(m)}^a, \tag{1}$$

together with the electric and magnetic Lorentz-force laws for the two charge-carrying fluids. The equations may be motivated by a convective matter-space variational construction, but the linear analysis below uses only the effective equations displayed in this section. Electric and magnetic charge are carried by two matter-space flows with identically conserved number currents, $\nabla_a n^a_{(I)} = 0$ ($I = e, m$), and fixed charge per particle q, g, so that

$$j^a_{(e)} = q\, n^a_{(e)}, \qquad j^a_{(m)} = g\, n^a_{(m)}. \tag{2}$$

The two fluids satisfy the electric and magnetic Lorentz-force laws

$$n^b_{(e)} \big(\mathrm{d}\mu^{(e)}\big)_{ba} = q\, n^b_{(e)} F_{ba}, \qquad n^b_{(m)} \big(\mathrm{d}\mu^{(m)}\big)_{ba} = g\, n^b_{(m)} {}^*F_{ba}. \tag{3}$$

The inertia and the inter-fluid entrainment follow from a Carter master function

$$\Lambda = -\rho_e(n_e) - \rho_m(n_m) - \frac{\alpha}{2} x^2, \qquad n_I = \sqrt{-n^a_{(I)} n_{(I)a}}, \quad x^2 = -n^a_{(e)} n_{(m)a}, \tag{4}$$

whose momenta $\mu^{(I)}_a = \partial\Lambda/\partial n^a_{(I)}$ take the entrained form

$$\mu^{(I)}_a = \mathcal{B}^{(I)} n^{(I)}_a + \mathcal{A}\, n^{(J)}_a, \qquad \mathcal{B}^{(I)} = \frac{1}{n_I}\frac{\mathrm{d}\rho_I}{\mathrm{d}n_I}, \quad \mathcal{A} = \frac{\alpha}{2}, \quad J \neq I. \tag{5}$$

The off-diagonal coefficient $\mathcal{A}$ is the inertial entrainment between the electric and magnetic fluids: upon linearization it supplies the off-diagonal entry ρ_{em} of the momentum matrix M introduced below, and hence the mixing parameter $\kappa^2 = \rho^2_{em}/(\rho_{ee}\rho_{mm})$, while $\alpha = 0$ decouples the fluids except through the field and gravity. When monopoles and antimonopoles are resolved separately, the same framework yields an exchange-symmetric pair whose intra-magnetic coupling β is likewise an ordinary Carter entrainment coefficient rather than an additional structure.

B. Linear response

The linearized Lorentz-force equations are

$$m_{(e)}\partial_t \mathbf{v}_{(e)} = q\,\mathbf{E}, \qquad m_{(m)}\partial_t \mathbf{v}_{(m)} = g\,\mathbf{B}. \tag{6}$$

Thus the electric fluid is accelerated by the electric field, while the magnetic fluid is accelerated by the magnetic field. For harmonic perturbations this gives

$$\mathbf{v}_{(e)} = \frac{iq}{m_{(e)}\omega}\mathbf{E}, \qquad \mathbf{v}_{(m)} = \frac{ig}{m_{(m)}\omega}\mathbf{B}, \quad \to \quad \mathbf{j}_{(e)} = \frac{i\omega^2_{pe}}{\omega}\mathbf{E}, \qquad \mathbf{j}_{(m)} = \frac{i\omega^2_{pm}}{\omega}\mathbf{B}, \tag{7}$$

where the electric and magnetic plasma frequencies,

$$\omega^2_{pe} = \frac{q^2 n^0_{(e)}}{m_{(e)}}, \qquad \omega^2_{pm} = \frac{g^2 n^0_{(m)}}{m_{(m)}}. \tag{8}$$

In a relativistic fluid these masses are replaced by the appropriate inertia per particle,

$$m_{(I)} \to h_I := \frac{\rho_I + p_I}{n_{(I)}}.$$

The two plasma frequencies are exchanged by electric–magnetic duality,

$$q \leftrightarrow g, \qquad n^0_{(e)} \leftrightarrow n^0_{(m)}, \qquad \omega_{pe} \leftrightarrow \omega_{pm}.$$

C. Transverse modes

We now consider transverse electromagnetic waves. Take the wave vector along the z-axis and choose a transverse polarization, $\mathbf{k} = k\hat{z}$, $\mathbf{E} = E\,\hat{x}$, $\mathbf{B} = B\,\hat{y}$. For perturbations proportional to $\exp[i(\mathbf{k}\cdot\mathbf{x} - \omega t)]$, the linearized Maxwell equations are

$$\nabla\times\mathbf{B} - \partial_t\mathbf{E} = \mathbf{j}_{(e)}, \qquad \nabla\times\mathbf{E} + \partial_t\mathbf{B} = -\mathbf{j}_{(m)}. \tag{9}$$

Using the linear response currents (7), these equations become

$$\left(\omega - \frac{\omega_{pe}^2}{\omega}\right) E = -kB, \qquad kE = -\left(\omega - \frac{\omega_{pm}^2}{\omega}\right) B. \tag{10}$$

The relative signs depend on the polarization convention, but the dispersion relation does not. Eliminating E and B gives

$$\boxed{\left(\omega^2 - \omega_{pe}^2\right)\left(\omega^2 - \omega_{pm}^2\right) = \omega^2 k^2} \quad \rightarrow \quad \omega^4 - \left(k^2 + \omega_{pe}^2 + \omega_{pm}^2\right)\omega^2 + \omega_{pe}^2\omega_{pm}^2 = 0. \tag{11}$$

Thus the two transverse branches are

$$\omega_\pm^2 = \frac{1}{2}\left[k^2 + \omega_{pe}^2 + \omega_{pm}^2 \pm \sqrt{\left(k^2 + \omega_{pe}^2 + \omega_{pm}^2\right)^2 - 4\omega_{pe}^2\omega_{pm}^2}\right]. \tag{12}$$

This expression is manifestly invariant under electric–magnetic duality, $\omega_{pe} \leftrightarrow \omega_{pm}$. The two transverse branches in

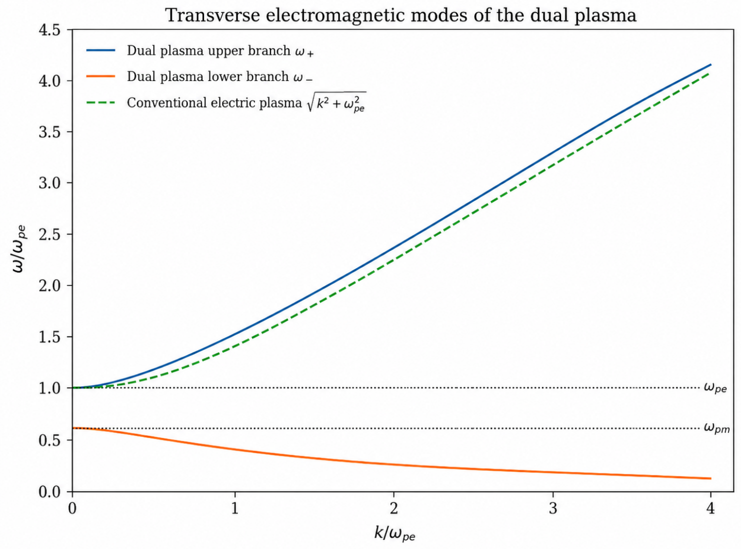

FIG. 1: Transverse electromagnetic spectrum of the dual plasma. The electric and magnetic plasma frequencies generate two duality-related cutoffs and two propagating transverse branches.

Eq. (12) are shown in Fig. 1. At zero wave number the two cutoffs are

$$\omega_+^2(0) = \max(\omega_{pe}^2, \omega_{pm}^2), \qquad \omega_-^2(0) = \min(\omega_{pe}^2, \omega_{pm}^2). \tag{13}$$

Thus the transverse response contains two plasma scales rather than one. If the magnetic plasma frequency is taken to zero, one recovers the usual electric plasma branch together with a collapsed zero-frequency branch,

$$\omega_+^2 = k^2 + \omega_{pe}^2, \qquad \omega_-^2 = 0 \qquad (\omega_{pm} = 0). \tag{14}$$

The extra branch is therefore not a relabeling of the ordinary electric plasma mode; it is the transverse dynamical response of the magnetic fluid.

The transverse sector is linearly stable. The discriminant satisfies

$$\left(k^2 + \omega_{pe}^2 + \omega_{pm}^2\right)^2 - 4\omega_{pe}^2\omega_{pm}^2 = k^4 + 2k^2(\omega_{pe}^2 + \omega_{pm}^2) + (\omega_{pe}^2 - \omega_{pm}^2)^2 \geq 0, \tag{15}$$

and both roots in Eq. (12) are non-negative. The upper branch approaches the vacuum light cone at large k,

$$\omega_+^2 = k^2 + \omega_{pe}^2 + \omega_{pm}^2 + O(k^{-2}), \tag{16}$$

whereas the lower branch becomes

$$\omega_-^2 = \frac{\omega_{pe}^2\omega_{pm}^2}{k^2} + O(k^{-4}). \tag{17}$$

The lower branch is therefore a medium-supported transverse mode. Its presence, together with the duality-related pair of cutoffs, is one of the simplest linear signatures that the dual plasma is physically distinct from conventional monopole magnetohydrodynamics.

D. Longitudinal modes without entrainment

In the absence of entrainment, the longitudinal electric and magnetic density oscillations decouple. For a longitudinal electric/magnetic perturbation, Gauss' law and charge conservation give the usual Langmuir mode,

$$\omega^2 = \omega_{pe}^2, \qquad \omega^2 = \omega_{pm}^2. \tag{18}$$

Thus the cold, non-entrained dual plasma contains two independent longitudinal plasma oscillations. This result should not be confused with conventional monopole magnetohydrodynamics, where the magnetic charge current is typically introduced phenomenologically into Maxwell's equations. Here the magnetic current is carried by an independent matter-space fluid and therefore has its own inertia, its own plasma frequency, and its own density mode.

The decoupling in this section is not protected once the two fluids are entrained. Since entrainment couples the momenta of the electric and magnetic constituents, it mixes precisely the longitudinal density oscillations described above. The resulting avoided crossing and stability boundary are derived in Sec. III.

E. Physical interpretation

The mode spectrum gives the first sense in which hidden magnetic charge can be dynamically visible. A neutral and unmagnetized background has no static Coulomb-like magnetic field, and therefore no direct monopole signal in the background electromagnetic field. Nevertheless, the magnetic fluid is not dynamically inert. Its inertia and charge enter the linear response through ω_{pm}, producing a second transverse cutoff, a second transverse branch, and a magnetic Langmuir mode.

This distinction will be important below. The absence of a static monopole field in a neutral background does not imply the absence of observable magnetic matter. Rather, the magnetic sector is probed through collective response. In the symmetric monopole–antimonopole plasma discussed in Sec. III C, this statement becomes sharper: net magnetic charge can be protected and strongly screened, while the hidden sector still leaves dynamical signatures in the wave spectrum and in the longitudinal electric–magnetic mixing.

III. ENTRAINMENT, PROTECTED NEUTRALITY, AND DYNAMICAL VISIBILITY

The transverse spectrum of Sec. II already shows that a dual plasma has more structure than a conventional monopole magnetohydrodynamic medium. We now turn to the part of the response where the two-fluid nature is most direct: the longitudinal sector. In a longitudinal perturbation both fluids move along the same wave vector, so any inertial coupling between the constituents enters without reference to polarization. This is the cleanest setting in which to isolate the effect of matter-space entrainment.

We parameterize the cold linearized momenta by

$$\mathbf{p}_{(I)} = \sum_J \mathsf{M}_{IJ}\mathbf{v}_{(J)}, \qquad \mathsf{M} = \begin{pmatrix} \rho_{ee} & \rho_{em} \\ \rho_{em} & \rho_{mm} \end{pmatrix}, \qquad \kappa^2 := \frac{\rho_{em}^2}{\rho_{ee}\rho_{mm}}. \tag{19}$$

Here ρ_{ee} and ρ_{mm} are the electric and magnetic inertial densities, while ρ_{em} is the entrainment coefficient. In a cold nonrelativistic limit one may identify

$$\rho_{ee} \simeq m_{(e)} n_{(e)}^0, \qquad \rho_{mm} \simeq m_{(m)} n_{(m)}^0,$$

whereas in a relativistic fluid the masses are replaced by the appropriate enthalpy per particle. The parameter κ measures the strength of off-diagonal inertial mixing. Absence of ghosts in the kinetic energy requires

$$\det \mathsf{M} > 0 \qquad \Longleftrightarrow \qquad \kappa^2 < 1. \tag{20}$$

In this section we focus on the longitudinal entrainment problem. The transverse branches derived in Sec. II C are therefore used as the unentrained electromagnetic reference spectrum. A fully transverse entrained response would require keeping the vector off-diagonal momentum response for both polarizations. That effect is not needed for the longitudinal stability and neutrality mechanism developed below.

A. Longitudinal mixing

Take a longitudinal plane wave with $\mathbf{k} = k\hat{z}$, $\mathbf{v}_{(e)} = v_{(e)}\hat{z}$, $\mathbf{v}_{(m)} = v_{(m)}\hat{z}$. The cold longitudinal force equations are

$$-i\omega \begin{pmatrix} p_{(e)} \\ p_{(m)} \end{pmatrix} = \begin{pmatrix} q n^0_{(e)} E_L \\ g n^0_{(m)} B_L \end{pmatrix}, \qquad \begin{pmatrix} p_{(e)} \\ p_{(m)} \end{pmatrix} = \begin{pmatrix} \rho_{ee} & \rho_{em} \\ \rho_{em} & \rho_{mm} \end{pmatrix} \begin{pmatrix} v_{(e)} \\ v_{(m)} \end{pmatrix}. \tag{21}$$

The longitudinal fields are fixed by the electric and magnetic Gauss laws. For harmonic perturbations, charge conservation gives

$$\delta n_{(e)} = \frac{k n^0_{(e)}}{\omega} v_{(e)}, \qquad \delta n_{(m)} = \frac{k n^0_{(m)}}{\omega} v_{(m)},$$

and hence the restoring forces are controlled by the two plasma frequencies

$$\omega^2_{pe} = \frac{q^2 \left(n^0_{(e)}\right)^2}{\rho_{ee}}, \qquad \omega^2_{pm} = \frac{g^2 \left(n^0_{(m)}\right)^2}{\rho_{mm}}. \tag{22}$$

Equivalently, after dividing by the diagonal inertias, the longitudinal eigenvalue problem may be written as

$$\begin{pmatrix} \omega^2 - \omega^2_{pe} & \kappa\,\omega^2 \\ \kappa\,\omega^2 & \omega^2 - \omega^2_{pm} \end{pmatrix} \begin{pmatrix} v_{(e)} \\ v_{(m)} \end{pmatrix} = 0, \tag{23}$$

where the sign of κ can be absorbed into the relative phase of the two velocity perturbations. The determinant condition gives

$$\boxed{(1-\kappa^2)\omega^4 - (\omega^2_{pe} + \omega^2_{pm})\omega^2 + \omega^2_{pe}\omega^2_{pm} = 0} \tag{24}$$

This reduces to the two independent Langmuir oscillations when $\kappa = 0$.

The two longitudinal eigenfrequencies are

$$\omega^2_{\pm,L} = \frac{\omega^2_{pe} + \omega^2_{pm} \pm \sqrt{(\omega^2_{pe} - \omega^2_{pm})^2 + 4\kappa^2 \omega^2_{pe}\omega^2_{pm}}}{2(1-\kappa^2)}. \tag{25}$$

Thus entrainment does not merely shift the plasma frequencies. It mixes the electric and magnetic density oscillations as shown in the $k = 0$ values in Fig. 2. Near resonance, $\omega_{pe} \simeq \omega_{pm}$, the eigenvectors are approximately the in-phase and out-of-phase combinations of the two fluids, and the frequency crossing is avoided. At exact resonance, $\omega_{pe} = \omega_{pm} = \omega_p$, one finds

$$\omega^2_{\pm,L} = \frac{\omega^2_p}{1 \mp \kappa}. \tag{26}$$

The splitting is therefore controlled directly by the entrainment strength.

Thermal pressure or finite compressibility dresses the two diagonal restoring forces. To leading order one may write

$$\omega^2_{pI} \longrightarrow \omega^2_{pI} + \sigma^2_I k^2, \qquad I = e, m, \tag{27}$$

where σ_I is the sound speed of species I. As shown in Fig. 2, this moves the location of the avoided crossing but does not change the ghost-free condition $\kappa^2 < 1$. If $\sigma_{(e)} \neq \sigma_{(m)}$, the crossing occurs near

$$k^2_* = \frac{\omega^2_{pe} - \omega^2_{pm}}{\sigma^2_{(m)} - \sigma^2_{(e)}} \tag{28}$$

when the right-hand side is positive.

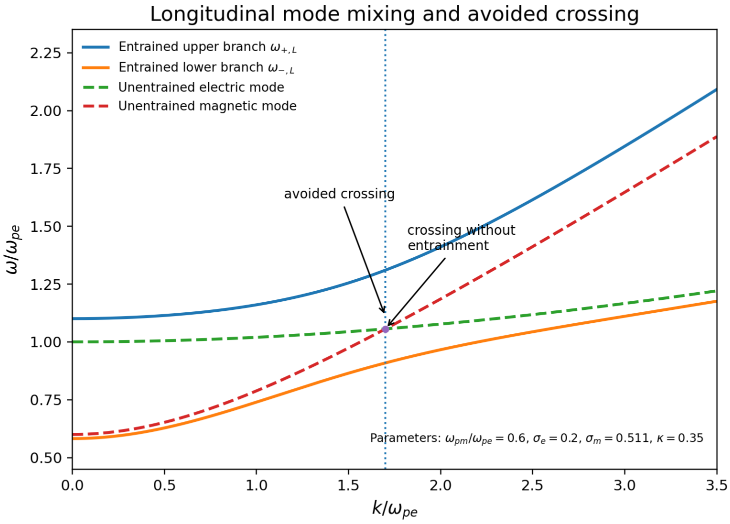

FIG. 2: Longitudinal electric–magnetic mode mixing in the presence of thermal pressure. Without entrainment the electric and magnetic plasma oscillations cross. Entrainment turns the crossing into an avoided crossing.

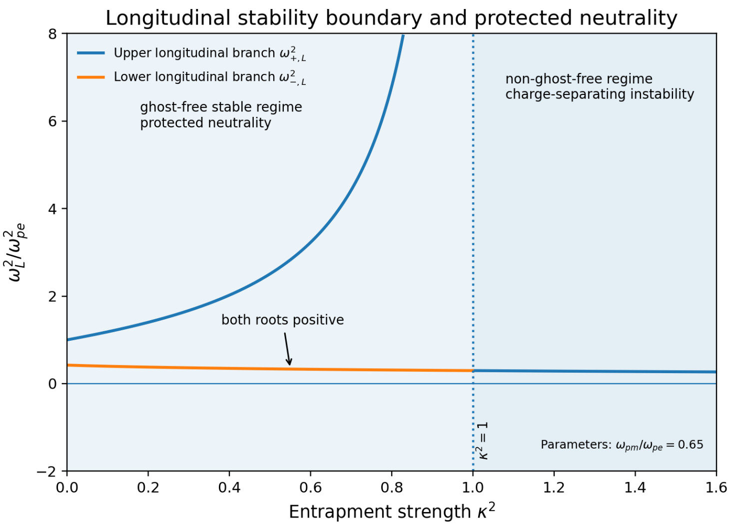

FIG. 3: Longitudinal stability boundary. In the ghost-free regime $\kappa^2 < 1$, both longitudinal modes have positive ω^2. The charge-separating instability appears only when the inertial matrix loses positive definiteness at $\kappa^2 = 1$.

B. Stability

The stability condition follows directly from Eq. (24). The product of the two roots is

$$\omega^2_{+,L}\omega^2_{-,L} = \frac{\omega^2_{pe}\omega^2_{pm}}{1-\kappa^2}. \tag{29}$$

For $\kappa^2 < 1$ this product is positive, and the sum of the two roots is also positive,

$$\omega^2_{+,L} + \omega^2_{-,L} = \frac{\omega^2_{pe} + \omega^2_{pm}}{1-\kappa^2} > 0. \tag{30}$$

Thus both longitudinal modes are stable in the ghost-free regime. For $\kappa^2 > 1$, the product of the roots is negative, so one mode has $\omega^2 < 0$. The onset of the longitudinal instability therefore coincides with the loss of positive definiteness of the inertial matrix:

$$\boxed{\text{longitudinal stability} \quad \Longleftrightarrow \quad \mathsf{M} > 0 \quad \Longleftrightarrow \quad \kappa^2 < 1.} \tag{31}$$

The schematic picture for this stability is given in Fig. 3.

This is a genuine two-fluid effect. The electric and magnetic fluids do not exchange charge, and the mixing is not generated by a direct conversion process. It is inertial: the momentum of each constituent contains a component proportional to the velocity of the other. Consequently the electric and magnetic Langmuir modes repel each other in the spectrum. This avoided crossing and its associated stability boundary are absent from conventional monopole MHD, where magnetic charge currents are introduced phenomenologically rather than as independent matter-space fluids with their own momenta.

C. Magnetic charge conjugation and protected neutrality

We now refine the magnetic sector by resolving it into monopole and antimonopole fluids. Let the two magnetic species be denoted $m+$ and $m-$, with independent matter spaces $\mathcal{M}_{(m+)}$ and $\mathcal{M}_{(m-)}$. Their number-current three-forms are $N^{(m+)}$ and $N^{(m-)}$, with dual current vectors $n^a_{(m+)}$ and $n^a_{(m-)}$. The two species carry opposite matter-space non-closures[1],

$$\mathrm{d}_{\mathcal{M}} G^{(m+)} = g\,\Omega^{(m+)}, \qquad \mathrm{d}_{\mathcal{M}} G^{(m-)} = -g\,\Omega^{(m-)}. \tag{32}$$

After pull-back,

$$J^{(m+)} = \mathrm{d}G^{(m+)} = gN^{(m+)}, \qquad J^{(m-)} = \mathrm{d}G^{(m-)} = -gN^{(m-)}. \tag{33}$$

The net magnetic current three-form and its dual vector are therefore

$$J^{(m)}_{\text{net}} = g\big(N^{(m+)} - N^{(m-)}\big), \qquad j^a_{(m),\text{net}} = g\big(n^a_{(m+)} - n^a_{(m-)}\big). \tag{34}$$

Now, we define magnetic charge conjugation by exchanging the two species together with their non-closure data,

$$\mathcal{C}_m : \qquad M^A_{(m+)} \leftrightarrow M^A_{(m-)}, \qquad G^{(m+)} \leftrightarrow G^{(m-)}. \tag{35}$$

Under this transformation, the net magnetic charge change sign:

$$N^{(m+)} \leftrightarrow N^{(m-)}, \qquad J^{(m)}_{\text{net}} \mapsto -J^{(m)}_{\text{net}}.$$

A $\mathcal{C}_m$-invariant background must therefore obey

$$N^{(m+)} = N^{(m-)}, \qquad j^a_{(m),\text{net}} = 0. \tag{36}$$

Net magnetic neutrality is then not an assumption imposed after the fact. It is the symmetry condition selecting the $\mathcal{C}_m$-even branch of the magnetic fluid.

To see whether this neutral branch is dynamically stable, consider a $\mathcal{C}_m$-invariant master function for the two magnetic species,

$$\Lambda_m = -\rho_m(n_{(m+)}) - \rho_m(n_{(m-)}) - \frac{\beta}{2}y^2, \qquad y^2 = -n^a_{(m+)} n_{(m-)a}, \qquad \beta \geq 0. \tag{37}$$

For a static homogeneous background, the magnetic contribution to the energy density is

$$E_m(n_{(m+)}, n_{(m-)}) = \rho_m(n_{(m+)}) + \rho_m(n_{(m-)}) + \frac{\beta}{2} n_{(m+)} n_{(m-)}. \tag{38}$$

At fixed total magnetic density

$$N = n_{(m+)} + n_{(m-)}, \qquad n_{(m\pm)} = \frac{N}{2} \pm \delta,$$

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[1] We barrow this form from the matter space framework. The two matter spaces $(m\pm)$ carry opposite magnetic charges.

one obtains

$$E_m = 2\rho_m(N/2) + \frac{\beta N^2}{8} + \left[\rho_m''(N/2) - \frac{\beta}{2}\right]\delta^2 + O(\delta^4). \tag{39}$$

The charge-conjugation symmetric point is therefore a local minimum if and only if

$$\boxed{2\rho_m''(N/2) - \beta > 0.} \tag{40}$$

The same condition is the ghost-free condition for the two magnetic species. Indeed, the Hessian of E_m at the symmetric point is

$$H = \begin{pmatrix} \rho_m'' & \beta/2 \\ \beta/2 & \rho_m'' \end{pmatrix}, \qquad \lambda_\pm = \rho_m'' \pm \frac{\beta}{2}. \tag{41}$$

The $\mathcal{C}_m$-breaking direction is the antisymmetric fluctuation $(1, -1)$, whose eigenvalue is

$$\lambda_- = \rho_m'' - \frac{\beta}{2}.$$

Thus the neutral branch is stable precisely when the antisymmetric mode is not a ghost. In the notation of Eq. (19), this may be written as

$$\kappa_{mm}^2 := \frac{(\beta/2)^2}{(\rho_m'')^2} < 1. \tag{42}$$

A spontaneous transition to a state with $J_{\rm net}^{(m)} \neq 0$ can occur only when this bound is violated.

Proposition 1 (Dynamical protection of magnetic neutrality)**.** *In the $\mathcal{C}_m$-invariant monopole–antimonopole fluid, the neutral background $N^{(m+)} = N^{(m-)}$ is energetically stable throughout the ghost-free regime. A charge-separating instability, and hence a nonzero net magnetic current $J_{\rm net}^{(m)}$, appears only when the antisymmetric longitudinal mode has already crossed the stability boundary.*

This is the precise sense in which net magnetic neutrality is protected. The theory does not require one to impose a vanishing monopole density by hand. Rather, the $\mathcal{C}_m$-even state is dynamically selected throughout the regime in which the effective two-fluid description has positive kinetic energy.

D. Screening, strong coupling, and thermal de-screening

Protected neutrality closes the direct static channel for observing a monopole plasma. In the $\mathcal{C}_m$-symmetric branch, the long-range $1/r^2$ field of the net magnetic charge cancels because

$$J_{\rm net}^{(m)} = 0.$$

This cancellation, however, should not be confused with the absence of magnetic matter. The individual $m+$ and $m-$ fluids still fluctuate, and their fluctuations are screened over the magnetic Debye length.

For a magnetic plasma with total magnetic density $n_{(m)}^0$, the magnetic plasma frequency is

$$\omega_{pm}^2 = \frac{g^2 n_{(m)}^0}{m_{(m)}}, \qquad n_{(m)}^0 = n_{(m+)}^0 + n_{(m-)}^0, \tag{43}$$

where the contributions of the two species add because screening depends on g^2, not on the sign of g. In a classical thermal regime, the magnetic Debye length

$$\lambda_{Dm}^2 = \frac{v_{th,m}^2}{\omega_{pm}^2} = \frac{T/m_{(m)}}{g^2 n_{(m)}^0/m_{(m)}} = \frac{T}{g^2 n_{(m)}^0} \propto T, \tag{44}$$

lengthens with temperature. Whether this screening still hides individual charges is controlled by the magnetic plasma coupling at the Wigner–Seitz radius $a_m = (n^0_{(m)})^{-1/3}$,

$$\Gamma_m = \frac{g^2/(4\pi a_m)}{T} = \frac{\alpha_m\,(n^0_{(m)})^{1/3}}{T}, \qquad N_D = n^0_{(m)}\lambda^3_{Dm} = \frac{T^{3/2}}{g^3(n^0_{(m)})^{1/2}} = (4\pi\Gamma_m)^{-3/2}\,, \tag{45}$$

as shown in Fig. 4, Dirac quantization gives a strong magnetic coupling,

$$\alpha_m = \frac{g^2}{4\pi} = \frac{1}{4\alpha} \simeq 34, \tag{46}$$

for the minimal Dirac charge in the convention $qg = 2\pi$. So neutrality and screening persist up to a temperature about 34 times the naive "Coulomb energy $\sim$ thermal energy" estimate: the strong coupling that enhances screening in Sec. III C also delays de-screening. For $\Gamma_m \gg 1$ ($N_D \ll 1$, low T) the medium is a strongly coupled, correlated neutral plasma: $\lambda_{Dm} \ll a_m$ and each monopole field is screened well inside the interparticle spacing—invisible. Because α_m is large, residual magnetic-charge fluctuations are strongly screened.

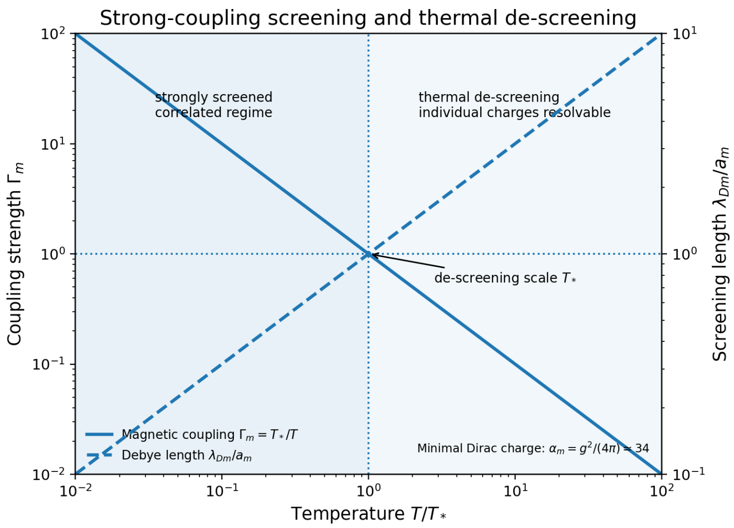

FIG. 4: Strong-coupling screening and thermal de-screening. Because the magnetic coupling is large, the monopole plasma remains strongly screened for $T < T_*$. Individual magnetic charges become thermally resolvable only above the de-screening scale.

Individual magnetic charges become thermally resolvable only when the temperature is high enough that the magnetic Debye length is comparable to, or larger than, the interparticle spacing. For $\Gamma_m \lesssim 1$ ($N_D \gtrsim 1$, high T) the plasma is weakly coupled: $\lambda_{Dm} \gtrsim a_m$, and over the window $a_m \lesssim r \lesssim \lambda_{Dm}$ the bare g/r^2 field of an individual charge is resolved. This gives the de-screening scale

$$\boxed{T_* \sim \alpha_m (n^0_{(m)})^{1/3} = \frac{g^2}{4\pi}(n^0_{(m)})^{1/3}.} \tag{47}$$

The large value of α_m therefore delays thermal de-screening: the static monopole channel remains hidden unless the temperature is parametrically high compared with the naive density scale $(n^0_{(m)})^{1/3}$.

Crossing T_* does not generate a net magnetic charge. The condition $J^{(m)}_{\rm net} = 0$ remains enforced by $\mathcal{C}_m$ as long as the symmetry is unbroken. What changes is the visibility of the individual $\pm g$ constituents. At low temperature, the plasma is strongly coupled and correlated, and the field of each charge is screened inside the interparticle scale. At high temperature, screening weakens and individual charges can be resolved over distances between a_m and λ_{Dm}.

E. Interpretation

The result is not that magnetic charge is simply unobservable. Rather, the static monopole signal is removed in the $\mathcal{C}_m$-symmetric branch, while the magnetic sector remains visible through collective response. The transverse spectrum contains a magnetic plasma scale, the longitudinal sector exhibits electric–magnetic mode mixing, and entrainment produces an avoided crossing and a stability boundary. These are dynamical signatures of the hidden magnetic fluid.

Thus the dual plasma realizes a separation between two notions of observability. As an isolated static source, net magnetic charge is screened or symmetry forbidden in the stable neutral branch. As a dynamical medium, however, the magnetic sector leaves measurable imprints in the electromagnetic response. The appropriate conclusion is therefore not absolute invisibility, but static hiding together with dynamical visibility.

F. Experimental observability

The ideal transverse cutoffs and sharp propagating branches derived above should not be taken as the most realistic experimental targets in ordinary spin ice. In spin-ice materials, where magnetic monopole excitations are realized effectively [15], monopole motion is strongly dissipative, and the clean collisionless dual-plasma dispersion is likely to be broadened into a relaxation spectrum. A more robust signature is instead the density dependence of the collective magnetic-charge response. Since the magnetic plasma scale is

$$\omega_{pm}^2 = \frac{g^2 n_{(m)}^0}{m_{(m)}},$$

or, more generally, with $m_{(m)}$ replaced by the appropriate effective inertia, the characteristic magnetic-charge relaxation or resonance frequency should scale with the monopole density as

$$\omega_{pm}^2 \propto n_{(m)}^0.$$

This scaling survives even when the ideal mode is damped, because it follows from the restoring force associated with magnetic charge density fluctuations rather than from the existence of an infinitely sharp quasiparticle pole.

Artificial spin ice provides a particularly promising arena for this test. In engineered two- or three-dimensional spin-ice architectures, the monopole density can be tuned by temperature, field history, or sample preparation, and the magnetic-charge configuration can be imaged directly. Microwave spectroscopy, Brillouin-light-scattering spectroscopy, or related dynamical probes could then be used to correlate the measured relaxation or resonance scale with the independently prepared monopole density. The cleanest evidence for the dual plasma response would not necessarily be a sharp transverse cutoff, but a systematic scaling of the collective magnetic-charge spectral feature according to

$$\omega_{\text{coll}}^2 \sim \omega_{pm}^2 \propto n_{(m)}^0.$$

Such an observation would distinguish a genuine magnetic-charge plasma response from a purely phenomenological monopole current, and would provide an experimentally accessible remnant of the dynamical visibility mechanism proposed here.

If the early universe selects the $\mathcal{C}_m$-symmetric branch, monopoles are produced with vanishing net magnetic charge but with a nonzero total magnetic density. In that case the relevant cosmological observable is not a long-range monopole field, since $J_{\text{net}}^{(m)} = 0$, but the collective magnetic plasma scale

$$\omega_{pm}^2(z) = \frac{g^2 n_{(m)}(z)}{m_{(m)}}.$$

This scale can modify the propagation and damping of primordial electromagnetic fields, and may leave imprints in the survival of primordial magnetic fields, CMB-era plasma transfer, or hidden-sector radiation dynamics. Thus a neutral monopole plasma would be cosmologically visible, if at all, through its density-dependent collective response rather than through a static monopole Coulomb field.

IV. DISCUSSION

We have studied the linear response of a dual plasma in which electric and magnetic charges are carried by independent matter-space fluids. The main conclusion is that magnetic charge need not be directly visible as an isolated static source in order to have observable consequences. In the stable neutral branch the long-range monopole field is absent or screened, yet the magnetic sector remains visible both through the collective dynamics of the medium and through the quantization of the electric charge it leaves behind.

There are three main dynamical results. First, the transverse electromagnetic spectrum contains two plasma scales: the electric and magnetic plasma frequencies enter symmetrically, giving two duality-related cutoffs and two transverse branches. The lower branch collapses to zero frequency when $\omega_{pm} \to 0$, showing that it is a genuine response of the magnetic component rather than a relabeling of the ordinary electric mode.

Second, the longitudinal sector exposes the two-fluid character of the theory. The cold non-entrained limit gives two independent Langmuir modes; finite compressibility dresses them by k^2-dependent corrections and allows them to cross; matter-space entrainment then mixes the two density oscillations and turns the crossing into an avoided crossing. The same inertial mixing yields a sharp stability condition $\kappa^2 < 1$, equivalent to positive definiteness of the momentum matrix. The avoided crossing and the threshold are therefore specific consequences of treating the magnetic carrier as an independent fluid with its own momentum, not phenomenological additions to monopole magnetohydrodynamics.

Third, resolving the magnetic component into monopole and antimonopole fluids shows how net magnetic neutrality is dynamically protected. In a $\mathcal{C}_m$-symmetric sector the neutral branch satisfies $J^{(m)}_{\rm net} = 0$; charge separation is the antisymmetric $m+-m-$ direction, and the onset of the charge-separating instability coincides with the loss of the ghost-free condition. The residual magnetic-charge fluctuations are Debye screened, with a screening strength enhanced by the large magnetic coupling $\alpha_m = g^2/4\pi \simeq 1/4\alpha$. This is an internal effective-field-theory mechanism for the long-range static invisibility of a neutral, strongly coupled monopole plasma, with a thermal de-screening scale $T_* \sim \alpha_m (n^0_{(m)})^{1/3}$ above which the individual $\pm g$ constituents become resolvable while the net charge stays zero.

Taken together, the dual plasma realizes a separation between three notions of observability. Statically, the channel is closed: there is no unscreened $1/r^2$ monopole field in the stable $\mathcal{C}_m$-symmetric branch. Dynamically, the channel is open: the magnetic plasma scale ω_{pm}, longitudinal electric–magnetic mixing, entrainment-controlled stability, and thermal de-screening are collective signatures of the hidden sector. Structurally, the hidden dyonic substructure leaves a permanent imprint in the quantization of electric charge, resolvable directly only above the binding scale. Magnetic charge is thus statically hidden, dynamically visible, and structurally imprinted.

From an experimental point of view, the clean ideal-plasma transverse cutoffs may not be the most robust target in ordinary spin ice, where monopole motion is strongly dissipative. A more realistic observable is the density dependence of a collective magnetic-charge relaxation or resonance scale. Since $\omega^2_{pm} = g^2 n^0_{(m)}/m_{(m)}$, or more generally with $m_{(m)}$ replaced by an effective inertia, the characteristic scale should obey $\omega^2_{\rm coll} \sim \omega^2_{pm} \propto n^0_{(m)}$. This scaling survives damping because it follows from the restoring force associated with magnetic-charge density fluctuations rather than from a sharp quasiparticle pole. Artificial spin ice, especially engineered two- or three-dimensional architectures, offers a natural arena: the monopole density can be prepared and imaged while microwave, Brillouin-light-scattering, or related probes test the predicted scaling.

The construction treats magnetic charge as an effective matter-space constituent. It does not assume a GUT monopole, nor does it fix the inertial mass: Dirac quantization constrains g, not the inertia in the plasma frequency, so the observable scale is $\omega^2_{pm} = g^2 n^0_{(m)}/m_{\rm eff}$ with $m_{\rm eff}$ the appropriate inertial parameter. The usual cosmological monopole problem concerns the abundance of relic magnetic charges [16, 17]; here the relevant question is instead whether a $\mathcal{C}_m$-symmetric population can be neutral in net magnetic charge while still carrying a collective plasma response. If the early universe approximately respects $\mathcal{C}_m$, monopoles and antimonopoles are produced symmetrically, so the symmetry suppresses the net asymmetry $J^{(m)}_{\rm net} \propto N^{(m+)} - N^{(m-)}$ rather than the total abundance $N^{(m+)} + N^{(m-)}$. A neutral monopole plasma then produces no long-range cosmological monopole field, but its density-dependent collective scale $\omega_{pm}(z)$ could modify the propagation or damping of primordial electromagnetic fields, or appear as a hidden-sector plasma response.

A further structural implication concerns electric charge quantization. The matter-space construction does not require a constituent to be purely electric or purely magnetic. A pair of dyons with opposite magnetic charges may form a magnetically neutral composite,

$$(q, g) + (q', -g) = (Q_{\rm net}, 0), \qquad Q_{\rm net} = q + q'.$$

Although such an object has no long-range monopole field, its internal two-body structure is not equivalent to two purely electric charges. The Dirac–Schwinger–Zwanziger pairing of the two constituents is

$$D_{12} = q(-g) - q'g = -Q_{\rm net} g.$$

The possibility of constituents carrying both electric and magnetic charge is standard in dyonic charge-lattice physics [3, 18, 19]. Quantum consistency requires $D_{12} \in 2\pi\mathbb{Z}$. For the minimal magnetic charge $g = 2\pi/e$, this gives

$$Q_{\rm net} \in e\mathbb{Z}.$$

Thus, if ordinary electric charges are interpreted as $\mathcal{C}_m$-even dyonic composites, their net magnetic charge is hidden while their surviving electric charge is quantized. This observation is not needed for the plasma-mode analysis above; it should be viewed instead as a possible structural imprint of the same hidden magnetic sector.

Several extensions are immediate. Magnetized backgrounds should reveal the dual analogues of cyclotron structure, Faraday rotation, and anisotropic wave propagation. Warm, kinetic, or dissipative treatments are needed to describe Landau damping, collisional broadening, and the relaxation spectra expected in effective monopole media such as spin ice. The same effective two-fluid equations can also be applied to relativistic or cosmological settings, including neutral monopole plasmas, hidden-sector charged media, and the propagation of primordial electromagnetic fields. Finally, the magnetically neutral dyonic interpretation mentioned above suggests a separate quantum problem. One should classify the admissible composites using the Dirac–Schwinger–Zwanziger charge lattice, including possible Montonen–Olive/Witten-type structures, and then analyze the binding dynamics of the corresponding dyonic states. These questions are not needed for the linear-response results derived here, and we leave them to future work.

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